Excitation spectra of non-uniform correlated many-body systems using linearised MPS

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Low-energy excitation spectra are a central tool for understanding correlated quantum many-body systems, yet their computation becomes difficult beyond mean-field theory and homogeneous settings. In this talk, I present an algebro-geometric approach to the tangent-space formulation of matrix product states (MPS) for the study of elementary excitations in finite, non-uniform one-dimensional lattice systems with open boundary conditions. The starting point is the familiar Gutzwiller ansatz, in which excitations are described as linearized fluctuations around a variational ground state. I show how this construction extends from separable product states to correlated MPS by interpreting excitations as tangent vectors to the MPS variational manifold, viewed as the smooth full-rank stratum of an algebraic variety cut out by tensor-rank, equivalently determinantal, conditions.

This perspective naturally connects the time-dependent variational principle with a linearized eigenvalue problem for collective modes. A key role is played by the decomposition of tangent directions into physical and gauge directions, which makes it possible to identify meaningful fluctuations and control redundancies arising from the MPS parametrization. From the algebro-geometric point of view, these structures reflect the local geometry of the MPS variety, its smooth locus, and its stratification by bond dimension. For numerical illustrations, I focus in particular on the one-dimensional Bose–Hubbard model.

More generally, the talk emphasizes that the MPS ansatz can be understood not only as a numerical variational class, but also as a geometric object with both differential- and algebraic-geometric structure. This viewpoint provides a bridge between tensor-network methods, variational dynamics, and algebraic geometry. It clarifies how correlated excitations emerge from the geometry of the MPS manifold and suggests new ways of analyzing variational spectra in strongly interacting quantum systems.